R8248

Sub. Code
511201

M.Sc. DEGREE EXAMINATION, APRIL – 2023

Second Semester

Mathematics

LINEAR ALGEBRA

(CBCS – 2022 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A $(10 \times 1 = 10)$

Answer **all** questions.

1. An $n \times n$ square matrix A over the field F is ———— if $A_{ij} = A_{ji}$ for each i and j.

   (a) Hermitian (b) Symmetric

   (c) Homogeneous (d) None of these

2. If V is a finite – dimensional vector space, then any two bases of V have ———— number of elements.

   (a) Different (b) Unknown

   (c) Same (d) Infinite

3. If A is an $m \times n$ matrix over the field F, the transpose of A is $n \times m$ matrix A^t defined by

   (a) $A_{ij}^{\ t} = A_{ij}$ (b) $A_{ji}^{\ t} = A_{ji}$

   (c) $A_{ij}^{\ t} = A_{ji}^{\ t}$ (d) $A_{ij}^{\ t} = A_{ji}$

4. A group is ———— if it satisfies the condition $xy = yx$ for each x and y.

   (a) Associative (b) Distributive

   (c) Commutative (d) None of these

5. The field F is algebraically closed if every prime polynomial over F has degree ————

(a) 2 (b) 3

(c) 0 (d) 1

6. A polynomial f of degree 'n' over a field F has atmost ———— roots in F.

(a) $n-2$ (b) n

(c) $n-1$ (d) n^2

7. If A' is a matrix obtained from A by interchanging two rows of A, then ————

(a) $D(A') = -D(A)$ (b) $D(A') = -D(A')$

(c) $D(A') = D(A)$ (d) None of these

8. A sequence $(K_1, K_2 \ldots K_n)$ of positive integers not exceeding n, with the property that no two of the K_i are equal, is called ———— of degree n.

(a) Permutation (b) Function

(c) Unique (d) Alternate

9. The collection of all α such that $T.\alpha = C.\alpha$ is ———— associated with C.

(a) Characteristic Roots

(b) Characteristic Space

(c) Characteristic Function

(d) Characteristic Vector

10. If N be a linear operator on the vector space V, then N is Nilpotent of some positive integer r such that ————

(a) $N^r = 1$ (b) $N^r = 0$

(c) $N^r = -1$ (d) $N^1 = N^r$

Part B $(5 \times 5 = 25)$

Answer **all** questions, choosing either (a) or (b).

11. (a) Define Vector space and their properties.

Or

(b) If V is a vector space over the field F, verify that $(\alpha_1 + \alpha_2) + (\alpha_3 + \alpha_4) = [\alpha_2 + (\alpha_3 + \alpha_1)] + \alpha_4$ for all vectors $\alpha_1, \alpha_2, \alpha_3$ and α_4 in V.

12. (a) Show that $F^{m \times n}$ is isomorphic to F^{mn}.

Or

(b) Let V be a finite-Dimensional vector space over the field F and let W be a subspace of V. Show that $\dim W + \dim W^{\circ} = \dim V$.

13. (a) Let F be a field. Show that the ideal generated by a finite number of polynomials $f_1, f_2, \ldots f_n$ in $F[x]$ is the intersection of all ideals containing $f, \ldots f_n$.

Or

(b) Suppose $f \equiv g \bmod p$ and $f_1 \equiv g_1 \bmod P$. Prove that

(i) $f + f_1 \equiv g + g_1 \bmod P$

(ii) $ff_1 \equiv gg_1 \bmod P$

14. (a) Let A be a 2×2 matrix over a field F and suppose that $A^2 = 0$. Show that for each scalar C that $\det(cI - A) = c^2$.

Or

(b) Let n be a positive integer and F a field. If σ is a permutation of degree n, prove that the function $T(x_1, x_2 \ldots x_n) = (x_{\sigma 1}, x_{\sigma_2}, \ldots x_{\sigma n})$ in an invertible linear operator on F^n.

15. (a) Let V be a finite-dimensional vector space and let W_1 be any subspace of V. Prove that there is a subspace W_2 of V such that $V = W_1 \oplus W_2$.

Or

(b) Let T be a Linear operator on V. If every subspace of V is invariant under T, then prove that T is a scalar multiple of the identity operator.

Part C $(5 \times 8 = 40)$

Answer any **five** questions.

16. If W_1 and W_2 are finite-dimensional subspaces of a vector space V, then show that $W_1 + W_2$ is finite dimensional and $\dim W_1 + \dim W_2 = \dim(W_1 \cap W_2) + \dim(W_1 + W_2)$.

17. Prove that the subspace spanned by a non-empty subset S of a vector space V is the set of all linear combinations of vectors in S.

18. Let m and n be positive integers and F a field. Let $f_1, \dots f_m$ be linear functionals on F^n. For α in F^n define $T_\alpha = (f_1(\alpha), \dots f_m(\alpha))$. Show that T is a linear transformation from F^n into F^m. Then show that every linear transformation from F^n into F^m is of the above form, for some $f_1, \dots f_m$.

19. If W_1 and W_2 are subspaces of a finite-dimensional vector space, then prove that $W_1 = W_2$ if and only if $W_1^0 = W_2^0$.

20. State and prove Taylor's Formula.

21. State and prove Cayley Hamilton Theorem.

22. Let K be a commutative ring with identity and let A and B be $n \times n$ matrices over K. Then show $\det(AB) = (\det A)(\det B)$.

23. Let V be a finite-dimensional vector space and let $W_1, \dots W_k$ be subspaces of V such that $V = W_1 + \dots W_K$ and $\dim V = \dim W_1 + \dots + \dim W_K$. Prove that $V = W_1 \oplus \dots \oplus W_K$.

R8250

Sub. Code
511203

M.Sc. DEGREE EXAMINATION, APRIL – 2023

Second Semester

Mathematics

COMPLEX ANALYSIS

(CBCS – 2022 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A $(10 \times 1 = 10)$

Answer **all** questions.

1. A rational function $R(z)$ of order p has p zeros and p poles, and every equation $R(z) = a$ has exactly.

   (a) p roots

   (b) $p + 1$ roots

   (c) $p - 1$ *roots*

   (d) None

2. The limit function of a uniformly convergent sequence of continuous functions is itself

   (a) convergence (b) constant

   (c) continuous (d) none

3. The integral of the function $\int_C e^z \cos z dz$ where C is the unit circle is

(a) $\frac{\pi}{2}(3+2i)$

(b) $\pi(3+2i)$

(c) $\frac{\pi}{3}(3+2i)$

(d) $\frac{\pi}{2}(2+3i)$

4. The converse of Cauchy- integral theorem is

(a) Euler's theorem

(b) Liouville's theorem

(c) Morera's theorem

(d) Goursat's theorem

5. A Maclaurin series is a Taylor series with center

(a) $z_0 = 0$ (b) $z_0 = 1$

(c) $z_0 = 2$ (d) $z_0 = -1$

6. The zero of the function $\frac{z}{\cos z}$ is

(a) 1 (b) -1

(c) 0 (d) π

7. Residue of the function $\frac{1}{z+z^2}$ at $z=0$ is

(a) 0 (b) 1

(c) –1 (d) 2

8. The zero of order is known as

(a) complex zero (b) simple zero

(c) singularity (d) none of these

9. The Laurent series expansion of the function $f(z)=\frac{1}{e^z-1}$ valid in the region $0<|z|<2$ is given by

(a) $f(z)=\frac{1}{z}-\frac{1}{2}+\frac{1}{3}z-\frac{1}{120}z^3+\cdots$

(b) $f(z)=\frac{1}{z}+\frac{1}{2}-\frac{1}{3}z+\frac{1}{120}z^3+\cdots$

(c) $f(z)=\frac{1}{z}-\frac{1}{2}+\frac{1}{12}z-\frac{1}{720}z^3+\cdots$

(d) $\frac{1}{z}-\frac{1}{2}+\frac{1}{12}z\frac{1}{120}z^3+\cdots$

10. In the Laurent expansion of $f(z)=\frac{1}{(z-1)(z-2)}$ valid in the region $1<|z|<2$, the coefficient of $\frac{1}{z^2}$ is

(a) 0 (b) $\frac{1}{2}$

(c) 1 (d) –1

Part B $(5 \times 5 = 25)$

Answer **all** questions, choosing either (a) or (b).

11. (a) If all zeros of a polynomial $P(z)$ lie in a half plane. then prove that all zeros of the derivative $p'(z)$ lie in the same half plane.

Or

(b) Define a linear transformation. Also prove that a linear transformation carries circles into circles.

12. (a) Derive the Cauchy's integral formula.

Or

(b) Suppose that $f(z)$ is analytic on a closed curve γ (i.e., f is analytic in a region that contains γ). Show that $\int_{\gamma} \overline{f(z)} f'(z) dz$ is purely imaginary. (The continuity of $f'(z)$ is taken for granted.)

13. (a) Prove that an analytic function comes arbitrarily close to any complex value in every neighborhood of an essential singularity.

Or

(b) State and prove the Schwarz's lemma.

14. (a) State and prove arguments principle.

Or

(b) Evaluate the following integrals by the method of residues

(i) $\int_0^{\infty} \frac{x^2 dx}{x^4 + 5x^2 + 6}$

(ii) $\int_0^{\infty} \frac{x^2 dx}{(x^2 + a^2)^3}$, a real

15. (a) Define an entire function with an example. Also Prove that every function which is meromorphic in the whole plane is the quotient of two entire functions.

Or

(b) Derive the Jensen's formula.

Part C $(5 \times 8 = 40)$

Answer any **five** questions.

16. If $\sum a_n z^n$ and $\sum b_n z^n$ have radii of convergence R_1 and R_2, show that the radius of convergence of $\sum a_n b_n z^n$ is at least $R_1 R_2$.

17. What is the general form of a rational function which has absolute value 1 on the circle $|z| = 1$? In particular, how are the zeros and poles related to each other?

18. State and prove the Cauchy's theorem for rectangle.

19. If the piecewise differentiable closed curve γ does not pass through the point a, then show the value of the integral $\int_\gamma \frac{dz}{\gamma z - a}$ is a multiple of $2\pi i$.

20. Derive Laurent series.

21. State and prove Taylor's theorem.

22. State and prove maximum principle for harmonic function.

23. Suppose that $f_n(z)$ is analytic in the region Ω_n, and that the sequence $\{f_n(z)\}$ converges to a limit function $f(z)$ in a region Ω, uniformly on every compact subset of Ω. Then prove $f(z)$ is analytic in Ω. Moreover, $f_n'(z)$ converges uniformly to $f'(z)$ on every compact subset of Ω.

R8251

Sub. Code
511204

M.Sc. DEGREE EXAMINATION, APRIL – 2023

Second Semester

Mathematics

PARTIAL DIFFERENTIAL EQUATIONS

(CBCS – 2022 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A $(10 \times 1 = 10)$

Answer **all** questions.

1. If $F(D)\,y = x^n$, n is a positive integer then

   (a) $P.I = [F(D)]^{-1}\{x^{-n}\}$

   (b) $P.I = [F(D)]\{x^{-n}\}$

   (c) $P.I = [F(D)]^{-1}\{x^{n}\}$

   (d) $P.I = [F(D)]\{x^{n}\}$

2. A necessary and sufficient condition that the pfaffian differential equation $X \cdot dr = 0$ should be ———— is that $X \cdot curlx = 0$.

   (a) differentiable (b) integrable

   (c) primitive (d) integrating factor

3. Eliminate orbitrary function form $z = f(x^2 + y^2)$, we get

(a) $py = qx$ (b) $px = qy$

(c) $\frac{y}{x} = \frac{p}{q}$ (d) $\frac{x}{y} = pq$

4. If $f(z) = \psi(x, y) + i\psi(x, y)$ the Cauchy-Riemann equation is satisfied.

(a) $\psi_x = -\psi_y, \psi_y = \psi_x$

(b) $\psi_x = \psi_y, \psi_y = -\psi_x$

(c) $\psi_x = -\psi_y, \psi_y = -\psi_x$

(d) $\psi_x = \psi_y, \psi_y = \psi_x$

5. If integrating the equation $\frac{dp}{p} = \frac{dq}{q}$ the compatible system is obtained as

(a) $q = ap$ (b) $\frac{1}{q} = \frac{1}{ap}$

(c) $\frac{1}{p} = \frac{1}{ap}$ (d) $p = aq$

6. A system of two first order partial differential equations of the form $f(x, y, u, p, q) = 0$ and $g(x, y, u, p, q) = 0$ are said to be

(a) Charpit's method

(b) Jacobi's method

(c) Compatible method

(d) Special types

7. Let a relation found by eliminating a and b between $\phi(x, y, z, a, b), \frac{d\phi}{da} = \frac{d\phi}{db} = 0$ is a called

(a) complete integral

(b) particular integral

(c) singular integral

(d) general integral

8. If $m_1 = m_2$ both the roots are real and equal, then C.F is

(a) $c_1 e^{m_1 x} + c_2 e^{m_2 x}$ (b) $(c_1 + c_2 x) e^{m_1 x}$

(c) both (a) and (b) (d) None of the above

9. The fundamental Maxwell's equation in the form $\nabla \times E_0$ is

(a) $\frac{1}{\mu} \frac{\partial H_0}{\partial t}$ (b) $\mu \frac{\partial H_0}{\partial t}$

(c) $\frac{-1}{\mu} \frac{\partial H_0}{\partial t}$ (d) $-\mu \frac{\partial H_0}{\partial t}$

10. The total flux of heat flow across S per unit time is given by

(a) $H = \iint_S (-k\nabla u) \cdot \hat{n} ds$

(b) $H = \iint_S (-k^2 \nabla u) \cdot \hat{n} ds$

(c) $H = \iint_S (-k\nabla^2 u) \cdot \hat{n} ds$

(d) $H = \iint_S (-k^2 \nabla^2 u) \cdot \hat{n} ds$

Part B $(5 \times 5 = 25)$

Answer **all** questions, choosing either (a) or (b).

11. (a) Find the integral curves of the sets of equations $\dfrac{dx}{xz-y} = \dfrac{dy}{yz-x} = \dfrac{dz}{1-z^2}$.

Or

(b) Prove that a pfaffian differential equation in two variable always possesses an integrating factor.

12. (a) Find the integral surface of the linear partial differential equation

$x(y^2+z)p - y(x^2+z)q = (x^2-y^2)z$ which contains the straight line $x+y=0,\ z=1$.

Or

(b) Find the general integrals of the linear partial differential equation.

$(y+zx)p-(x+yz)q = x^2-y^2$

13. (a) Show that the equation $xp = yq$, $z(xp+yq) = 2xy$ are compatible and solve them.

Or

(b) Find complete integral $xy + 3yq = 2(z - x^2q^2)$.

14. (a) Solve $(y^3x - 2x^4)p + (2y^4 - x^3y)q = qz(x^3 - y^3)$.

Or

(b) Find the particular integral of

$(D^2 - 2DD' + D'^2)z = 12xy$.

15. (a) Solve $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ for $0 < x < \pi, 0 < \pi$, $0 < y < \pi$ under the boundary condition

$u(x, 0) = x^2, u(x, \pi) = 0,$

$ux(0, y) = \frac{\partial}{\partial x} u_x(0, y) = 0 = u_x(\pi, y)$

Or

(b) Write down the derivation for two-dimensional wave equation.

Part C (5 × 8 = 40)

Answer any **five** questions.

16. Verify that the differential equation $(y^2 + yz)\, dz + (xz + z^2) dy + (y^2 - xz)\, dy = 0$ is integrable and find its primitive.

17. Find the integral surface of the equation $xp + yq = z$ passing through $x + y = 1$ and $x^2 + y^2 + z^2 = 4$.

18. Solve $x^3 \frac{d^3 y}{dx^3} + 3x^2 \frac{d^2 y}{dx^2} + x \frac{dy}{dx} + 8y = 65 \cos(\log x)$.

19. Find the surface which is orthogonal to one parameter system $z = cxy(x^2 + y^2)$ and passes through the hyperbola $x^2 - y^2 = a^2$, $z = 0$.

20. Determine the complete integral of the equation $p^2 + q^2 - 2px - 2qy + 1 = 0$.

21. Find the laplace transform of the following function

(a) $F(t) = t \sin at$

(b) $F(t) = t \cos at$

22. Find the solution of the equation $\frac{\partial}{\partial r}\left(r^2 \frac{\partial \phi}{\partial r}\right) + \frac{1}{\sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta \frac{d\phi}{d\theta}\right) = 0$ in the form $\phi = f(r)\cos\theta$, given that

(a) $\frac{-\partial \phi}{\partial r} = u\cos\theta$ when $r = a$

(b) $\frac{-\partial \phi}{\partial r} = 0$ when $r = \infty$

23. Derive the general solution of three-dimensional diffusion equation.

R8252

Sub. Code
511510

M.Sc. DEGREE EXAMINATION, APRIL 2023

Second Semester

Mathematics

Elective : INTRODUCTION TO PYTHON PROGRAMMING

(CBCS – 2022 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A $(10 \times 1 = 10)$

Answer **all** questions.

1. A program must be converted to ————— language to be executed by a computer.

   (a) Assembly (b) Machine

   (c) High Level (d) Very high level

2. This license allows a patent grant for derivative works

   (a) BSD License (b) Apache License

   (c) MIT License (d) CC license

3. The variable defined outside the function is referred as

   (a) Static (b) global

   (c) Automatic (d) register

4. The data type of svs.argv is —————

   (a) Set (b) list

   (c) Tuple (d) strings

5. The error that is not a standard exception in Python is ____________

   (a) Name error

   (b) Assignment error

   (c) IO error

   (d) Value error

6. The set of statements that will be executed whether an exception is thrown or not?

   (a) Except

   (b) else

   (c) Finally

   (d) Assert

7. The basic ndarray is created using?

   (a) numpy.array(object,dtype = None, copy = True. subok = False, ndmin = 0)

   (b) numpy.array(object,dtype = None, copy = True, order = None, subok = False, ndmin = 0)

   (c) numpy_array(object, dtype = None, copy = True, order = None, subok = False, ndmin = 0)

   (d) numpy.array(object. dtype = None, copy = True, order = None, ndmin = 0)

8. What will be output for the following code?

   import numpy as np

   a=np.array([1,2,3])

   print a

   (a) [[1,2. 3]] (b) [1]

   (c) [1.2.3] (d) Error

9. What will be the output of the following Python code?

```
i = 1
while True;
    if i%3 = 0
        break
    print(i)
i+=1
```

(a) 1 2 3 (b) error

(c) 1 2 (d) 1 3

10. Python supports the creation of anonymous functions at runtime, using a construct called ————

(a) pi (b) anonymous

(c) lambda (d) beta

Part B $(5 \times 5 = 25)$

Answer **all** questions, choosing either (a) or (b).

11. (a) Differentiate between Interpreter and Compiler.

Or

(b) Mention disadvantages of Assembly language.

12. (a) Differentiate between local and global variables with suitable examples.

Or

(b) Write a program using functions to display Pascal's triangle.

13. (a) Explain the syntax of the try – except – finally block.

Or

(b) Write a program that uses a while loop to add up all the even numbers between 100 and 200.

14. (a) Explain sorting iterable in Python with example.

Or

(b) Differentiate between NumPy and SciPy in Python.

15. (a) Write a program in Python to find GCD of two or more integers.

Or

(b) Write a program in Python to find prime numbers for the given integers.

Part C (5 × 8= 40)

Answer any **five** questions.

16. Write a sort note on data types in Python.
17. Explain the utility of doc strings.
18. Briefly explain the conditional statements available in Python.
19. Explain Type conversion in Python with example.
20. Write a program in Python to find the mean, median, mode and standard deviation for the given integers.
21. Write a program in Python to find the product of the two matrices.
22. Demonstrate how functions return multiple values with example.
23. Differentiate between syntax error and an exception.

———————